CBCS SCHEME

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17MAT31

Third Semester B.E. Degree Examination, Dec.2023/Jan.2024
Engineering Mathematics - III

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $0 \le x \le 2\pi$. Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$ **(08 Marks)**

b. Express $f(x) = 1 - (x/\ell)$ as a half range cosine series in $0 < x < \ell$. **(06 Marks)**

c. Express Y as a Fourier series upto first harmonic from the following table:

x	0	π/3	2π/3	π	4π/3	5π/3	2π
y	7.9	7.2	3.6	0.5	0.9	6.8	7.9

(06 Marks)

OR

2 a. Expand $f(x) = 2x - x^2$ as a Fourier series in $0 \le x \le 2$. **(08 Marks)**

b. Find the Fourier series for the function $f(x) = \begin{cases} -1 + x, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{cases}$ **(06 Marks)**

c. Expand Y in a Fourier series first harmonic

x	0	1	2	3	4	5	6
y	4	8	15	7	6	2	4

(06 Marks)

Module-2

3 a. Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$. Hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$ **(08 Marks)**

b. Find the Fourier, sine transform of $\frac{e^{-ax}}{x}, a > 0$. **(06 Marks)**

c. Solve the difference equation $u_{n+2} - 3u_{n+1} + 2u_n = 0$. Given $U_0 = 0$, $U_1 = 1$. **(06 Marks)**

OR

4 a. Find the Fourier cosine transform of $f(x) = \begin{cases} 4x, & 0 \le x \le 1 \\ 4 - x & 1 < x \le 4 \\ 0 & x > 4 \end{cases}$ **(08 Marks)**

b. Find the Z-transform of Cosn θ. **(06 Marks)**

c. Find the inverse Z-transform of $\frac{2z^2 + 3z}{(z-2)(z-4)}$. **(06 Marks)**

Module-3

5 a. Compute the coefficient of correlation and the equation of the lines of regression for the following data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(08 Marks)

b. Fit a curve of the form $y = a\,e^{bx}$ by the method of least squares for the following data :

x	0	2	4
y	8.12	10	31.82

(06 Marks)

c. Find the real root of the equation $3x = \cos x + 1$ correct to four decimal places, using Newton Raphson method near $x = 0.5$. (06 Marks)

OR

6 a. If θ is the acute angle between the lines of regression, then show that

$\tan\theta = \dfrac{\sigma_x\sigma_y}{\sigma x^2 + \sigma y^2}\left(\dfrac{1-r^2}{r}\right)$. Explain the significance when $r = 0$ and $r = \pm 1$ (08 Marks)

b. Fit a parabola $y = a + bx + cx^2$ by the method of least squares for the following data.

x	0	1	2
y	1	6	17

(06 Marks)

c. Use the Regula-falsi, method to obtain $2x - \log_{10} x = 7$, which lies between 3.5 and 4, correct to two decimal places. (06 Marks)

Module-4

7 a. The population of a town is given by the table:

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

Using Newton's interpolation formula, calculate the increase in the population form the year 1955 to 1985. (08 Marks)

b. Use Lagrange's interpolation formula to find y at x = 10 given

x	5	6	9	11
y	12	13	14	16

(06 Marks)

c. Use Simpson's 1/3rd rule with seven ordinates to evaluate $\int_2^8 \dfrac{dx}{\log_{10} x}$ (06 Marks)

OR

8 a. Fit an interpolation polynomial for the data :

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

by using Newton's divided difference formula and hence find f(8) (08 Marks)

b. Applying Lagrange's formula inversely and find a root of the equation f(x) = 0 given that f(30) = –30, f(34) = –13, f(38) = 3, f(42) = 18. (06 Marks)

c. Evaluate $\int_0^1 \dfrac{dx}{1+x^2}$ by Weddle's rule taking seven ordinates. (06 Marks)

Module-5

9 a. Verify Green's theorem for $\oint_C (xy + y^2)dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(08 Marks)**

b. Verify Stoke's theorem for $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **(06 Marks)**

c. Prove that geodiscs of a plane are straight lines. **(06 Marks)**

OR

10 a. Using Gauss divergence theorem, evaluate $\iint_s (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot \hat{n}\, dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant. **(08 Marks)**

b. Find the extremal of the functional of the functional $\int_{x_1}^{x_2} (y' + x^2 y'^2)dx$ **(06 Marks)**

c. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

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